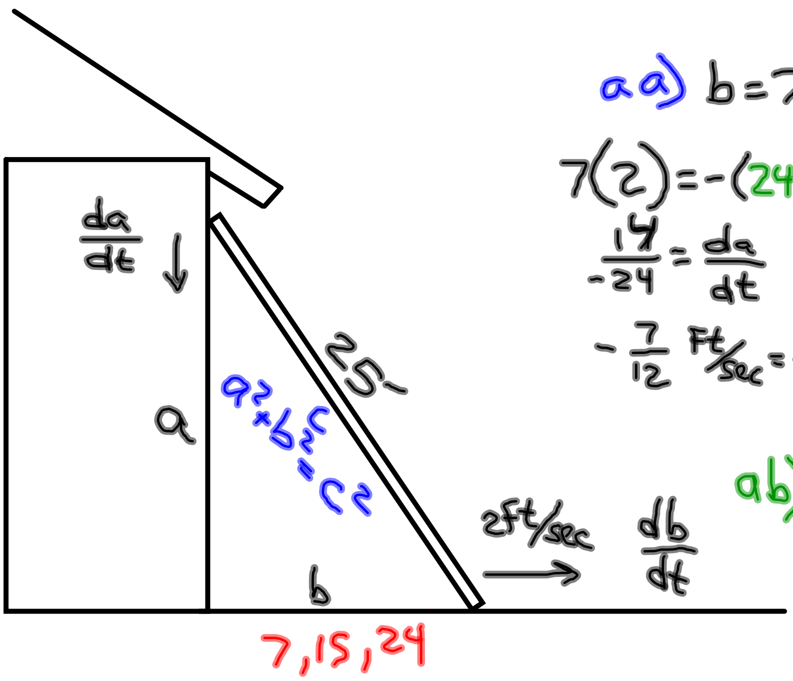
27) A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet/sec.

a) How fast is the top of the ladder moving down the wall when the base of the ladder is 7, 15, and 24 feet from the wall?



aa) $b = 7'$

$a^2 + b^2 = 25^2$

$a^2 + b^2 = 625$

$\cancel{2}a\frac{da}{dt} + \cancel{2}b\frac{db}{dt} = 0$

$b\frac{db}{dt} = -a\frac{da}{dt}$

$7(2) = -(24)\frac{da}{dt}$

$\frac{14}{-24} = \frac{da}{dt}$

$-\frac{7}{12}$ ft/sec $= \frac{da}{dt}$

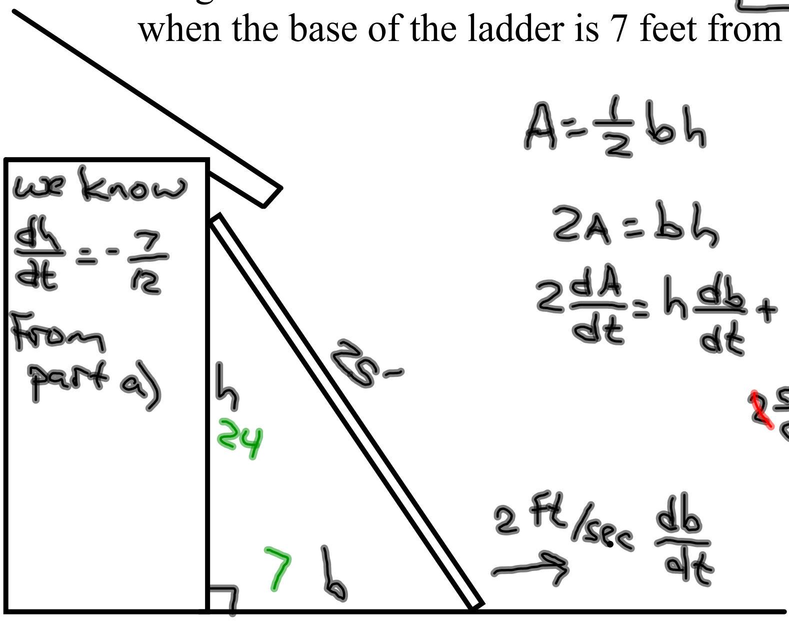
ab) $b = 15$ $\quad 15(2) = -(20)\frac{da}{dt}$

ac) $b = 24$ $\quad 24(2) = -(7)\frac{da}{dt}$

27) A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet/sec.

b) Consider the triangle formed by the side of the house, the ladder,and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 7 feet from the wall.

$$\frac{dA}{dt}$$



$$A = \frac{1}{2}bh$$

$$2A = bh$$

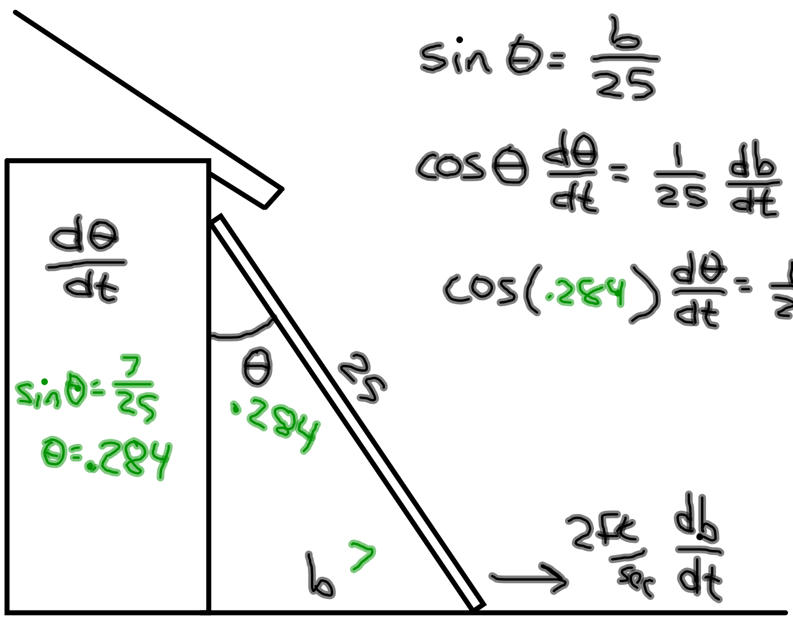
$$2\frac{dA}{dt} = h\frac{db}{dt} + b\frac{dh}{dt}$$

$$\cancel{2}\frac{dA}{dt} = (24)\cancel{2} + \left(\frac{7}{2}\right)\left(\frac{-7}{12}\right)$$

$$\frac{dA}{dt} = \frac{24^2}{24} - \frac{49}{24} = \boxed{\frac{527}{24}\ \frac{ft^2}{sec}}$$

27) A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet/sec.

c) Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 7 feet from the wall.



$\sin\theta = \frac{b}{25}$

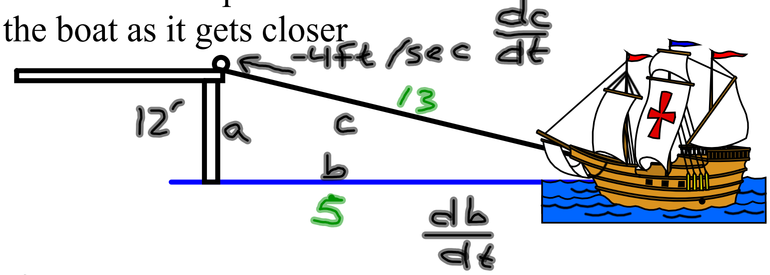
$\cos\theta \frac{d\theta}{dt} = \frac{1}{25}\frac{db}{dt}$

$\cos(.284)\frac{d\theta}{dt} = \frac{1}{25}(2)$

$\frac{rad}{sec}$

30) A boat is pulled into a dock by means of a winch 12 ft above the deck of the boat.

a) The winch pulls in rope at a rate of 4 ft/sec. Determine the speed of the boat when there is 13 ft of rope out. What happens to the speed of the boat as it gets closer to the dock?



$a^2+b^2=c^2$

$144+b^2=c^2$

$0+\cancel{2}b\frac{db}{dt}=\cancel{2}c\frac{dc}{dt}$

$5\frac{db}{dt}=13(-4)$ $\frac{db}{dt}=-\frac{52}{5}$ ft/sec

b) Suppose the boat is moving at a constant rate of 4 ft/sec. Determine the speed at which the winch pulls in rope when there is a total of 13 ft of rope out. What happens to tthe speed at which the winch pulls in rope as the boat gets closer to the dock?

$b\frac{db}{dt}=c\frac{dc}{dt} \rightarrow 5(4)=13\frac{dc}{dt} \Rightarrow \frac{20}{13}\frac{ft}{sec}=\frac{dc}{dt}$

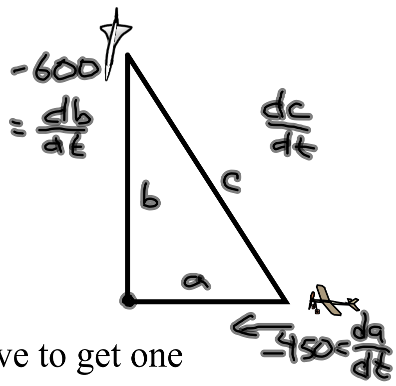
31) An air traffic controller spots two planes at the same altitude converging on a point as they fly at right angles to each other. One plane is 150 mi from the point moving at 450 mph. The other plane is 200 miles from the point moving at 600 mph.

a) At what rate is the distance betweeen the planes decreasing?

$$c^2 = a^2 + b^2$$

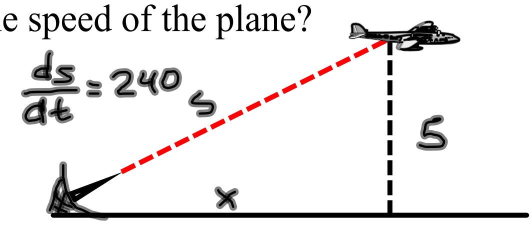
$$\cancel{2}c\frac{dc}{dt} = \cancel{2}a\frac{da}{dt} + \cancel{2}b\frac{db}{dt}$$

$$(250)\frac{dc}{dt} = (150)(-450) + (200)(-600)$$



b) How much time does the air traffic controller have to get one of the planes on a different flight path?

32) An airplane is flying at an altitude of 5 mi and passes directly over a radar antenna. When the plane is 10 mi away, the radar detects that the distance s is changing at a rate of 240 mph. What is the speed of the plane?



$$x^2 + 5^2 = s^2$$

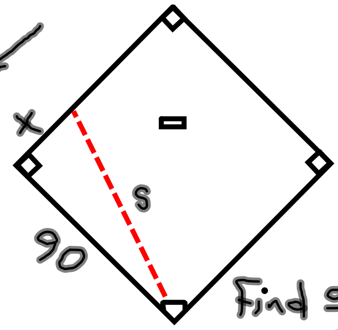
$$\cancel{2}x\frac{dx}{dt} + 0 = \cancel{2}s\frac{ds}{dt}$$

$$(5\sqrt{3})\frac{dx}{dt} = 10(240)$$

$$\frac{dx}{dt} = \boxed{\frac{2400}{5\sqrt{3}}\ \text{mi/hr}}$$

33) A baseball diamond has the shape of a square with sides 90 ft long. A player running from second to third at a speed of 28 ft/sec is 30 ft from third. At what rate is the player's distance *s* from home plate changing?

$\frac{dx}{dt} = -28$



Find $\frac{ds}{dt}$

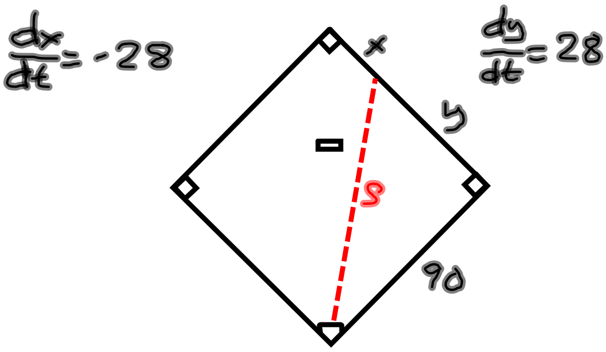
$x^2 + 90^2 = s^2$

$\cancel{2}x\frac{dx}{dt} = \cancel{2}s\frac{ds}{dt}$

Pythagorean Theorem

$30(-28) = (94.868)\frac{ds}{dt}$

34) Using the baseball diamond, suppose the player is running from first to second at a rate of 28 ft/sec. Find the rate at which the distance from home plate is changing when the player is 30 ft from second base?



Find $\frac{ds}{dt}$